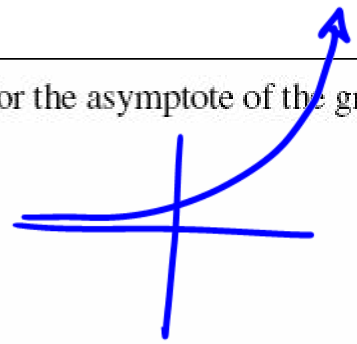
Logarithmic and Exponential Functions Review

Part A. No calculators allowed.

1. Determine an equation for the asymptote of the graph of $y = 2^{x+3} + 4$.



7.2

A. $y = 4$

~~B. $x = 3$~~

~~C. $x = -3$~~

D. $y = -4$

2. Solve: $9^x = 27^{x-3}$

7.3

A. -9

B. 3

C. $\frac{9}{2}$

D. 9

$(3^2)^x = (3^3)^{x-3}$

$2x = 3x - 9$

$x = 9$

3. Solve: $\log_5(3x) - \log_5(x-3) = 2$

8.4a

A. -6

B. $-\frac{1}{2}$

C. $\frac{75}{22}$

D. 11

$\log_5\left(\frac{3x}{x-3}\right) = 2$

$\frac{25}{1} = \frac{3x}{x-3}$

$25x - 75 = 3x$

$22x = 75$

4. If $\log_2 5 = x$ and $\log_2 3 = y$, determine an expression for $\log_2\left(\frac{15}{2}\right)$, in terms of x and y.

8.3

A. xy

B. $x + y$

C. $xy - 1$

D. $x + y - 1$

$\log_2 15 - \log_2 2$

$\log_2(5 \cdot 3) - 1$

$\log_2 5 + \log_2 3 - 1$

$x + y - 1$

$(\log_2 5)(\log_2 3)$

5.	Evaluate: $\log_5 \sqrt{5^3} = x$ 8.1 A. $\frac{1}{6}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 6 $5^x = \sqrt{5^3}$ $= (5^3)^{\frac{1}{2}}$ $= 5^{3/2}$
6.	Solve: $\log_2\left(\cancel{\log_x(x+6)}\right) = 1$ $\log_2 a = 1$ 8.1 A. 2 B. 3 C. 2, 3 D. −2, 3 $2^1 = \log_x(x+6)$ $x^2 = x+6$ $x^2 - x - 6 = 0$ $(x-3)(x+2) = 0$ $x = 3$ or $\cancel{-2}$ can't have a negative base
7.	Determine the domain of $y = \log(x+1)$. 8.1 A. $x < 1$ B. $x > 1$ C. $x < -1$ D. $x > -1$ $x+1 > 0$ $x > -1$

8.

Determine an equivalent expression for $\log \frac{100a^2}{\sqrt{b}}$.

8.3

A. $2\log 100a - \frac{1}{2}\log b$

B. $2 + 2\log a - \frac{1}{2}\log b$

C. $4\log a - \frac{1}{2}\log b$

D. $100 + 2\log a - \frac{1}{2}\log b$

$\log 100a^2 - \log\sqrt{b}$

$\log 100 + \log a^2 - \log b^{1/2}$

$\log_{10} 100 + 2\log a - \frac{1}{2}\log b$

$2 + 2\log a - \frac{1}{2}\log b$

9.

Evaluate: $\log_{\sqrt{7}} 7^3$

8.1

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. 6

D. 9

$\sqrt{7}^{x} = 7^3$

$(7^{\frac{1}{2}})^6 = 7^3$

10.

As an iceberg melts during the summer, it loses 3% of its mass every 5 days. This iceberg reduces to 40% of its original mass after t days. Which equation could be used to determine the value of t?

A. $40 = 100(0.97)^{\frac{t}{5}}$

numerator is actual amount of time, not part of rate

B. $40 = 100(0.97)^{\frac{5}{t}}$

C. $\cancel{40 = 100(1.03)^{\frac{t}{5}}}$

D. $\cancel{40 = 100(1.03)^{\frac{5}{t}}}$

$2^{-2} = \frac{1}{4}$

11. Solve: $\log_2\left(\log_9 x\right) = -1$

8.1

A. $\frac{1}{81}$

B. $\frac{1}{3}$

C. 3

D. 81

$2^{-1} = \log_9 x$

$\frac{1}{2} = \log_9 x$

$9^{\frac{1}{2}} = x$

12. Solve: $5^{x+1} = 2\left(3^{2x}\right)$

8.4

~~A.~~ $x = \frac{-\log 5}{1 - 2\log 6}$

~~B.~~ $x = \frac{-\log 5}{\log 5 - 2\log 6}$

C. $x = \frac{\log 2 - \log 5}{1 - 2\log 3}$

D. $x = \frac{\log 2 - \log 5}{\log 5 - 2\log 3}$

$\log 5^{x+1} = \log(2 \cdot 3^{2x})$

$(x+1)(\log 5) = \log 2 + 2x \log 3$

$x\log 5 + \underline{\log 5} = \underline{\log 2} + \underline{2x\log 3}$

$x\log 5 - 2x\log 3 = \log 2 - \log 5$

$x(\log 5 - 2\log 3) = \log 2 - \log 5$

13. Change $\log_2(3x) = 5$ to exponential form.

A. $3x = 2^5$

B. $3x = 5^2$

C. $2 = (3x)^5$

D. $2 = 3x^5$

$2^5 = 3x$

Part B. Calculators allowed.

14. A population grows continuously according to the formula $P = P_0 e^{kt}$, where P is the final population at the end of t years, P_0 is the initial population, and k is the annual growth rate. What will the population be at the end of 10 years if the initial population is 5000 and the annual growth rate is 3%?

A. 6 720
B. 6 749
C. 51 523
D. 100 428

$5000\, e^{(.03)(10)}$

$=$

$\ln = \log_e$

$\ln e^4 \stackrel{?}{=} \log_e e^4$

15. In 1872, Washington State experienced an earthquake of magnitude 6.8 on the Richter scale. Determine the magnitude on the Richter scale of an earthquake that is half as intense as the Washington State earthquake.

A. ~~3.4~~
B. ~~6.0~~
C. 6.5
D. ~~7.1~~

$10^{6.8} = 6310000$

$= 3155000$

16. Change to logarithmic form $a^3 = b$.

A. $3 = \log_a b$
B. $3 = \log_b a$
C. $b = \log_a 3$
D. $a = \log_b 3$

$\log_a b = 3.$

17. A population grows continuously according to the formula $P = P_0 e^{kt}$, where P is the final population at the end of t years, P_0 is the initial population and k is the annual growth rate. What will the population be at the end of 8 years if the initial population is 15 million and the annual growth rate is 4%?

A. 20.66 million
B. 124.90 million
C. 179.02 million
D. 367.99 million

$= 15\, e^{(.04 \times 8)}$

$=$

18.	Determine the magnitude of an earthquake that is half as intense as an earthquake of magnitude 8.0 on the Richter scale. A. 4.0 B. 5.0 C. 7.7 ~~D. 8.3~~

$10^8 = 100\,000\,000$

$= 50\,000\,000$

Part C. Written. Show all necessary work.

Solve algebraically $\log 2 - \log(x-1) = \log(x+1) - \log(x+17)$.

$$\frac{2}{x-1} = \frac{x+1}{x+17}$$

$$2x+34 = x^2-1$$

$$0 = x^2-2x-35$$

$$= (x-7)(x+5)$$

$$x = 7 \text{ or } \cancel{-5}$$

In a population of moths, 78 moths increase to 1000 moths in 40 weeks. What is the doubling time for this population of moths?

(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

$$y = y_0(a)^{\frac{t}{n}}$$

$$\frac{1000}{78} = \frac{78(2)^{\frac{40}{n}}}{78}$$

$$\frac{1000}{78} = 2^{40/n}$$

$$\frac{\log\left(\frac{1000}{78}\right)}{\log 2} = \frac{\left(\frac{40}{n}\right)\log 2}{\log 2}$$

$$\boxed{\frac{3.680}{1} = \frac{40}{n}}$$

$$3.68n = 40$$

$$n = \frac{40}{3.68} = 10.9$$