8.4B Warmup

1. The pH of lemon juice is 2.2, while the pH of coffee is 4.8. How many times more acidic is lemon juice than coffee?

difference = 2.6

$10^{2.6} = 398.1$

lemon juice is 398.1 × more acidic

2. Solve for B in terms of A and C

a) $\underbrace{\log 2B - \log 3C}$ = A

$\log\left(\frac{2B}{3C}\right) = A$

$\frac{3C}{2} \cdot 10^A = \frac{2B}{3C} \cdot \frac{3C}{2}$

$\boxed{B = \frac{3C}{2} \cdot 10^A}$

b) $\underbrace{\log A + \log B} = \log C$

$\log(AB) = \log C$

$AB = C$

$B = \frac{C}{A}$

3. If $a = \log 3$ and $b = \log 5$, express each of the following in terms of a and b

a) $\log\sqrt{\frac{5}{3}} = \log\left(\frac{5}{3}\right)^{1/2}$

$= \frac{1}{2}\log\left(\frac{5}{3}\right)$

$= \frac{1}{2}(\log 5 - \log 3)$

$= \frac{1}{2}(b - a)$

b) $\log\frac{27}{2500} = \log 27 - \log 2500$

$\log 3^3 - (\log 25 + \log 100)$

$\log 3^3 - \log 5^2 - \log 10^2$

$3\log 3 - 2\log 5 - 2\log 10$

$\boxed{3a - 2b - 2}$

4. Which of the following are identities? $(A, B, C > 0 \quad B \neq 1)$ Base change formula.

a) $\underbrace{\log_B A} = -\log_{\frac{1}{B}} A$

$\frac{\log A}{\log B}$ ✓

$= -\frac{\log A}{\log \frac{1}{B}}$

$= \frac{\log A}{-\log \frac{1}{B}}$

$= \frac{\log A}{\log\left(\frac{1}{B}\right)^{-1}} = \frac{\log A}{\log B}$ ✓

b) $(\log_B C)(\log_C A) = \log_B A$

$\frac{\cancel{\log C}}{\log B} \cdot \frac{\log A}{\cancel{\log C}} = \frac{\log A}{\log B}$

$\frac{\log A}{\log B}.$

Exponential Equations

Exponential equations which cannot be converted to the same base can be solved by using logarithms. Express the value of *x* in terms of logs and as a decimal to 2 decimal places.

1. $8^{x+1} = 20$

$$\log(8^{x+1}) = \log 20$$

$$\frac{(x+1)\cancel{\log 8}}{\cancel{\log 8}} = \frac{\log 20}{\log 8}$$

$$x+1 = \frac{\log 20}{\log 8}$$

$$x = 1.4406 - 1 = 0.4406$$

$\underline{\underline{\text{or}}}$ $(x+1)\log 8 = \log 20$

$$x\log 8 + \log 8 = \log 20$$

$$x\log 8 = \log 20 - \log 8$$

$$x = \frac{\log 20 - \log 8}{\log 8}$$

2. $20 = 6(2)^{-0.3x}$

$$\log 20 = \log(6 \cdot 2^{-.3x})$$

$$\log 20 = \log 6 + \log 2^{-.3x}$$

$$\frac{\log 20 - \log 6}{(-.3\log 2)} = \frac{-\cancel{.3}x\,\cancel{\log 2}}{-\cancel{.3}\,\cancel{\log 2}}$$

$$x = -5.79.$$

3. $5^{x+1} = 2^{x-3}$

$$\log 5^{x+1} = \log 2^{x-3}$$

$$(x+1)\log 5 = (x-3)\log 2$$

$$x\log 5 + \log 5 = x\log 2 - 3\log 2$$

* GCF

$$\log 5 + 3\log 2 = x\log 2 - x\log 5$$

$$= x(\log 2 - \log 5)$$

$$\frac{\log 5 + 3\log 2}{(\log 2 - \log 5)} = x$$

$$x = -4.03$$

4. $2(7)^{x-2} = 3(5)^{3x}$

$$\log(2\cdot 7^{x-2}) = \log(3\cdot 5^{3x})$$

$$\log 2 + \log 7^{x-2} = \log 3 + \log 5^{3x}$$

$$\log 2 + x\log 7 - 2\log 7 = \log 3 + 3x\log 5$$

$$\log 2 - 2\log 7 - \log 3 = 3x\log 5 - x\log 7$$

$$= x(3\log 5 - \log 7)$$

$$\frac{\log 2 - 2\log 7 - \log 3}{3\log 5 - \log 7} = x$$

$$x = -1.49$$

5. Solve for x: $p\,a^x = n^{x-1}$

P412 #2,7 allparts

$$\log(p\cdot a^x) = \log(n^{x-1})$$

$$\log p + \log a^x = \log n^{x-1}$$

$$\log p + x\log a = (x-1)(\log n)$$

$$= x\log n - \log n$$

$$\log p + \log n = x\log n - x\log a$$

$$= x(\log n - \log a)$$

$$\frac{\log p + \log n}{\log n - \log a} = x$$

6. How long will it take for money invested at 5% compounded monthly to double in value?

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$2 = 1\left(1+\frac{.05}{12}\right)^{12t}$$

$$\log 2 = \log\left(1+\frac{.05}{12}\right)^{12t}$$

$$\frac{\log 2}{\log\left(1+\frac{.05}{12}\right)} = \frac{12t\log\left(1+\frac{.05}{12}\right)}{\log\left(1+\frac{.05}{12}\right)}$$

$$166.7 = 12t$$

$$\frac{12t}{12} = \frac{166.7}{12}$$

$$t = 13.89 \text{ years}$$

7. The half-life of plutonium-239 is about 25 000 years. How many years does it take until only 36% of the plutonium still remains?

$$Y = Y_0 (a)^{t/n}$$

$$36 = 100(.5)^{\frac{t}{25000}}$$

$$\log 36 = \log\left(100(.5)^{t/25000}\right)$$

$$\log 36 = \log 100 + \frac{t}{25000}\log .5$$

$$\log 36 - \log 100 = \frac{t}{25000}(\log .5)$$

$$\frac{\log 36 - \log 100}{\log .5} = \frac{t}{25000}$$

$$\frac{t}{25000} = 1.474$$

$$\boxed{t = 36848 \text{ years}}$$

8. It is estimated that 20% of a certain radioactive substance decays in 30 hours. What is the half-life of the substance?

$$Y = Y_0 (a)^{\frac{t}{n}}$$

$$80 = 100(0.5)^{\frac{30}{n}}$$

$$.8 = 0.5^{\frac{30}{n}}$$

$$\log .8 = \frac{30}{n}\log .5$$

$$\frac{\log .8}{\log .5} = \frac{30}{n}$$

cross multiply + divide

n = 93.2 hours.

$\boxed{\text{p 412 \#11–15, 18}}$