8.4A Equations Involving Logarithms

To solve an equation involving logarithms, either

Express the equation as a single log equaling a single log: $\log_B X = \log_B Z \quad \Rightarrow \quad X = Z$

Or

Convert the equation to a single log equaling a number: $\log_B X = N \quad \Rightarrow \quad B^N = X$

It is also important to be aware of the restrictions on the variable in the equation. Apparent solutions may sometimes be extraneous because they fail to meet the restrictions. Alternately, you can check each solution in the equation to make sure that it is valid.

Example 1: Solve for x:

	Restriction
a) $\log_2(x-3)+7=8$ $-7 \quad -7$ $\log_2(x-3)=1$ $2^1 = x-3$ $x=5$ ✓ 1) check by looking at restriction or 2) check by substitution	$\log_2(x-3)$ $x-3>0$ $x>3$ 5 is larger than 3 so is a valid solution for x $\log_2(5-3) \rightarrow \log_2(2)$
b) $\log_5(x^2-5)=\log_5(x+1)$ quadratic $x^2-5=x+1$ $x^2-x-6=0$ $(x-3)(x+2)=0$ $x=3$ or $\cancel{-2}$ -2 is an "extraneous root"	$\log_5(3^2-5)$ ✓ $\log_5(3+1)$ ✓ $\log_5((-2)^2-5)$ not possible to log(−ve) ∴ reject -2 as a solution

c) $\log_2(x-2)+\log_2 x=\log_2 3$

$$\log_2[(x-2)(x)] = \log_2 3$$

$$\log_2(x^2-2x) = \log_2 3$$

$$x^2-2x = 3$$

$$x^2-2x-3=0$$

$$(x-3)(x+1)=0 \qquad x=3 \text{ or } \cancel{-1}$$

reject $x=-1$

a) $\log_2(-1) \leftarrow$ not possible

<u>or</u>

restriction on x is
$x-2>0$ <u>and</u> $x>0$

d) $\log_4(2x+2)-\log_4(3x+1)=\frac{1}{2}$

$$\log_4\left(\frac{2x+2}{3x+1}\right) = \frac{1}{2}$$

$$4^{\frac{1}{2}} = \frac{2x+2}{3x+1}$$

$$\frac{2}{1}\cdot 3x+1 = \frac{2x+2}{\cancel{3x+1}}\cdot \cancel{3x+1}$$

cross multiply.

$$6x+2 = 2x+2 \qquad 4x=0$$

$\boxed{x=0}$ ✓

e) $\log_5(3x+1)=3-\log_5(x-3)$

$$+\log_5(x-3) \quad +\log_5(x-3)$$

$$\log_5[(3x+1)(x-3)] = 3$$

$$5^3 = (3x+1)(x-3)$$

$$125 = 3x^2-9x+x-3$$

$$0 = 3x^2-8x-128$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2-4(3)(-128)}}{2(3)}$$

$$x=8 \text{ or } \cancel{-5\tfrac{1}{3}}$$

reject $x=-5\frac{1}{3}$
because
$\log_5(-5\frac{1}{3}-3)$
we get a
log of a
negative

f) $\log_2(x-3)-2=\log_2\left(\frac{1}{x}\right)$

$\log_2(x-3)-\log_2\left(\frac{1}{x}\right)=2$

$\log_2\left(\frac{x-3}{\left(\frac{1}{x}\right)}\right)=2$

$\frac{(x-3)}{\left(\frac{1}{x}\right)}=2^2$

$(x)(x-3)=2^2$

$x^2-3x=4$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4$ ✓ or $\cancel{-1}$

g) $5^{\log_5(2x-8)}=12$

① $\log_5$ and $5^{\square}$ cancel out.

$\cancel{5}^{\cancel{\log_5}(2x-8)}=12$

$2x-8=12$

$2x=20$

$x=10$

② convert the exp. form to log form

$\log_5 12=\log_5(2x-8)$

$12=2x-8$

Example 2: Determine the inverse of $y=5\log_4(2x-7)+3$

switch x and y

isolate log first.

$x=5\underbrace{\log_4(2y-7)}+3$

$-3 \qquad\qquad -3$

$\frac{x-3}{5}=\frac{\cancel{5}\log_4(2y-7)}{\cancel{5}}$

$\frac{x-3}{5}=\log_4(2y-7)$

$4^{\left(\frac{x-3}{5}\right)}=2y-7$

$2y=4^{\left(\frac{x-3}{5}\right)}+7$

$y=\frac{4^{\left(\frac{x-3}{5}\right)}+7}{2}$

p412

#1,3-6, 8-10