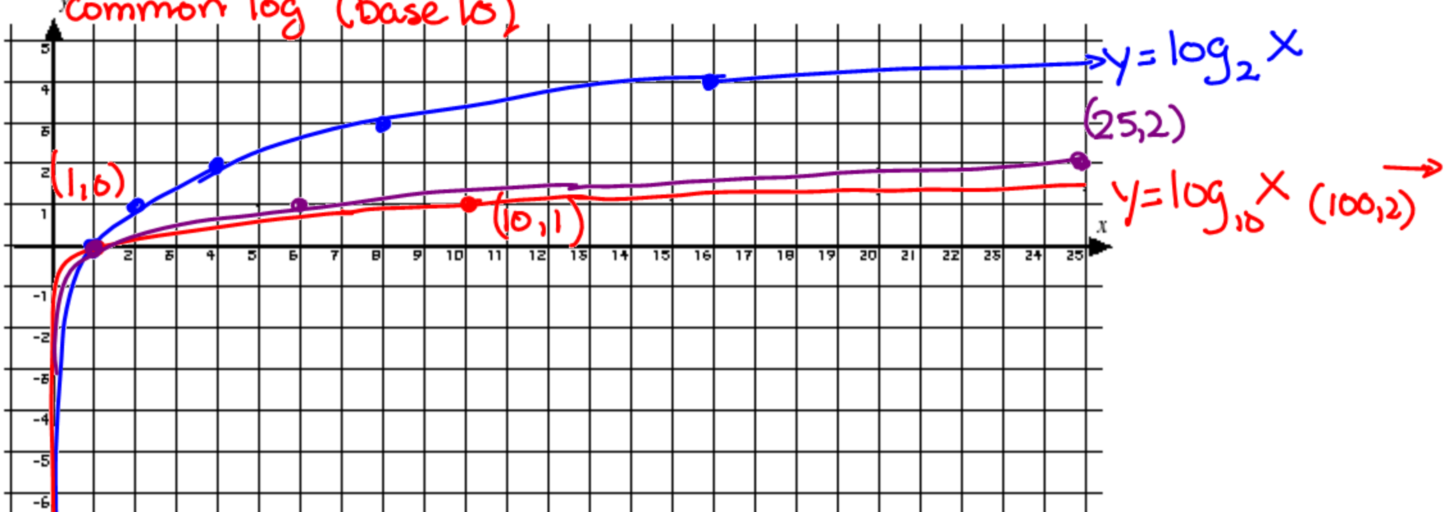
8.2 Transformations of Logarithmic Functions

Determine 5 ordered pairs that the function $y = \log_2 x$ passes through, and then sketch a graph of the function. To easily find ordered pairs that work, either

1) Think of the inverse of this function, and generate ordered pairs for it	2) Choose "nice" values for x – in this case you would use powers of 2	3) Choose values for y instead of x, and then think of the exponential form of each statement
$y = 2^x$ $(0,1) \rightarrow (1,0)$ $(1,2) \rightarrow (2,1)$ $(2,4) \rightarrow (4,2)$ $(3,8) \rightarrow (8,3)$ $(4,16) \rightarrow (16,4)$ $y = \log_2 x$	$y = \log_2 x$ x | y 4 | $2 \leftarrow \log_2(4)$ 16 | 4 32 | $5 \leftarrow \log_2(32)$	$y = \log_2 x$ x | y 16 | $4 \leftarrow 4 = \log_2 x$ $2^4 = x$ 1 | $0 \leftarrow 0 = \log_2 x$ $2^0 = x$

Graph $\boxed{y = \log x}$ and $\boxed{y = \log_2 x}$ on the same set of coordinate axes.

common log (base 10)



note: vertical asymptote at $x = 0$

What is the likely position of $y = \log_5 x$? Sketch $y = \log_5 x$ as well.

Sandwiched inbetween $\log_2 x$ and $\log_{10} x$

Example 1. Write the equation of the function $y = \log_2 x$ following the transformations indicated.

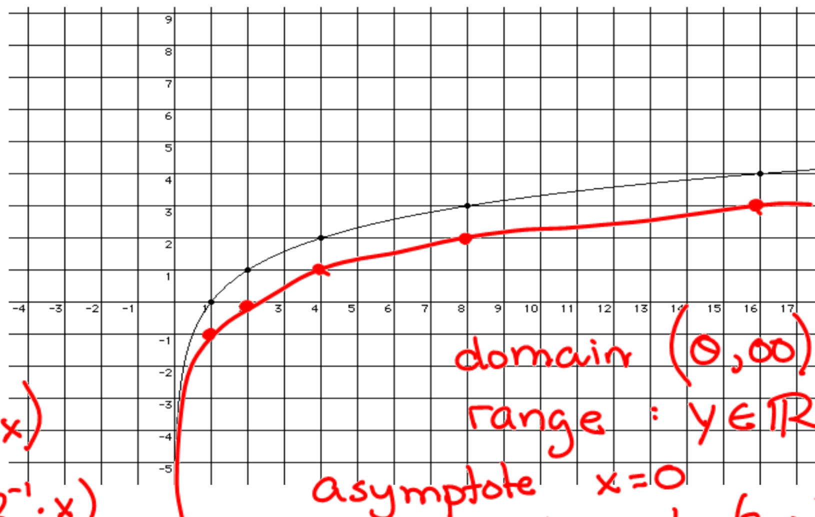
a) Translating 2 units right $x \to x-2$	$y = \log_2(x-2)$
b) Vertically expanding by 3, and reflecting in the *x*-axis	$y = -3\log_2 x$
c) Horizontally compressing the graph by one half	$y = \log_2(2x)$
d) Translating the graph 4 units down and reflecting in the *y*-axis	$y = \log_2(-x) - 4$
e) Translating 2 units down, then 3 units left and then expanding vertically by a factor of 5 and reflecting in the *y*-axis	$y = 5(\log_2(\cancel{x}-x+3) - 2)$

$y = 5\log_2(-x+3) - 10$

Example 2. For each of the following, indicate the new coordinates of the transformed points, and then sketch the resulting graph. State the domain and range of the transformed function, the *x*-intercept, and the equations of any asymptotes.

a) $y = \log_2 0.5x$

$(0.5, -1) \to$	$(1, -1)$
$(1, 0) \to$	$(2, 0)$
$(2, 1) \to$	$(4, 1)$
$(4, 2) \to$	$(8, 2)$
$(8, 3) \to$	$(16, 3)$



domain $(0, \infty)$

range : $y \in \mathbb{R}$

asymptote $x = 0$

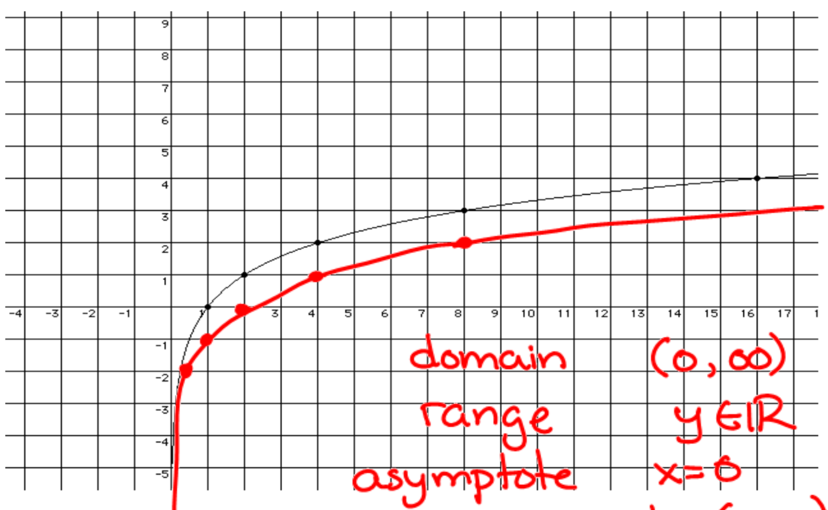
x-intercept : $(2, 0)$

equivalent to $y = \log_2\left(\frac{1}{2} \cdot x\right)$

$= \log_2(2^{-1} \cdot x)$

b) $y = \log_2 x - 1$ down 1

$(0.5, -1) \to$	$(0.5, -2)$
$(1, 0) \to$	$(1, -1)$
$(2, 1) \to$	$(2, 0)$
$(4, 2) \to$	$(4, 1)$
$(8, 3) \to$	$(8, 2)$



domain $(0, \infty)$

range $y \in \mathbb{R}$

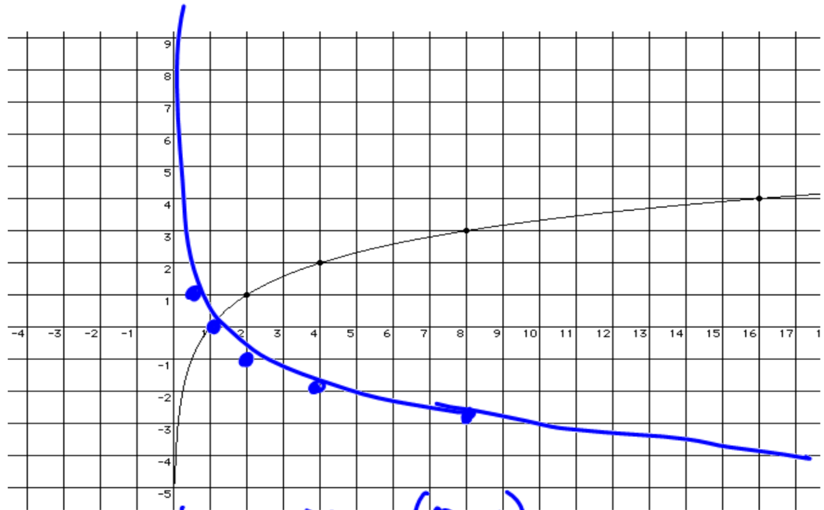
asymptote $x = 0$

x-int : $(2, 0)$

$y = \log_2 x - 1$ is the same as $y = \log_2(2^{-1}x)$

c) $y = -\log_2 x$

$(0.5,-1) \rightarrow$	$(0.5, 1)$
$(1, 0) \rightarrow$	$(1, 0)$
$(2, 1) \rightarrow$	$(2, -1)$
$(4, 2) \rightarrow$	$(4, -2)$
$(8, 3) \rightarrow$	$(8, -3)$



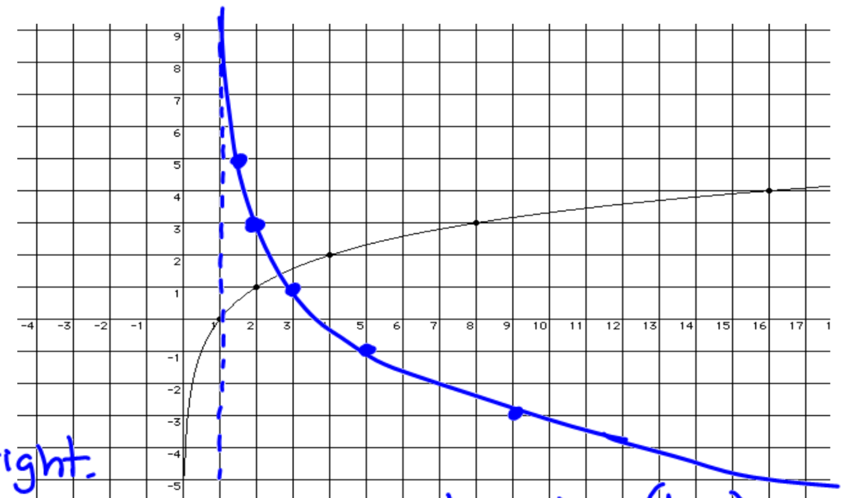
domain $(0, \infty)$

range $y \in \mathbb{R}$

asymptote: $x = 0$ x-int $= (1, 0)$

d) $y = -2\log_2(x-1)+3$

$(0.5,-1) \rightarrow$	$(1.5, 5$
$(1, 0) \rightarrow$	$(2, 3$
$(2, 1) \rightarrow$	$(3, 1$
$(4, 2) \rightarrow$	$(5, -1$
$(8, 3) \rightarrow$	$(9, -3$



asymptote: moves 1 right.

$x = 1$

$0 = -2\log_2(x-1)+3$

$\frac{-3}{-2} \div \log_2(x-1)$

$2^{\frac{3}{2}} = x - 1$

$x = 2^{1.5} + 1$

domain: $(1, \infty)$

range: $y \in \mathbb{R}$

asymptote: $x = 1$

x-int at $(3.83, 0)$

Example 3. Analyze the graph of $y = \log(2x+3)$. Identify the domain, range, asymptotes and intercepts.

$y = \log_{10} x$

translation left 3 then stretch by $\frac{1}{2}$

y-int

$y = \log(2 \cdot 0 + 3)$

$y = \log(3)$

x-int

$0 = \log(2x+3)$

$10^0 = 2x + 3$

$1 = 2x + 3$

$\boxed{x = -1}$

asymptote

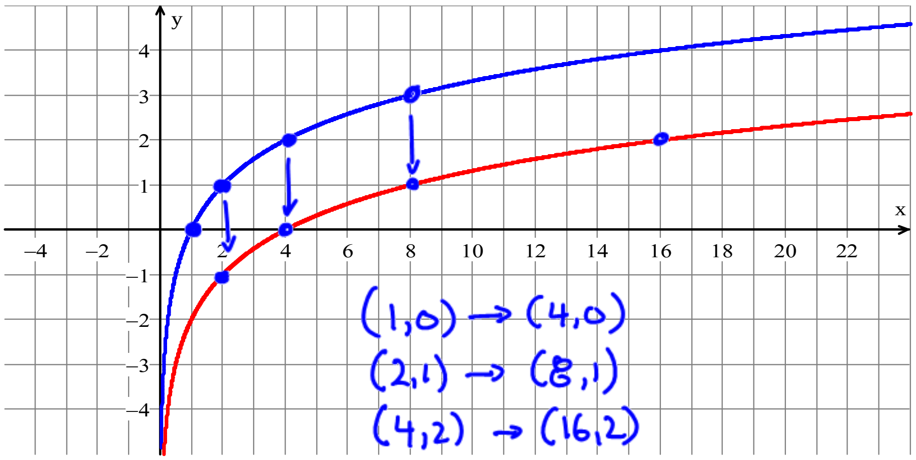
$x = 0 \rightarrow x = -3 \rightarrow x = -1\frac{1}{2}$

asymptote $x = -1.5$

domain $x > -1.5$

range: $y = \mathbb{R}$.

Example 4. What transformation could be done to the graph of $y = \log_2 x$ to produce the other graph given? In this case, there are at least two possible answers.



translating down 2.

$y = \log_2 x \;-2$

horizontal stretch ×4

$y = \log_2\left(\frac{1}{4}x\right)$

Example 5. Some scientists have discovered that there is a logarithmic relationship between butterflies and flowers. The relationship $F = -2.641 + 8.958\log B$ was found between the number, *F*, of flower species that a butterfly feeds on and the number, *B*, of butterflies observed. Predict the number of butterfly observations in a region with 30 flower species.

$$\overbrace{30}^{y_1} = \overbrace{-2.641 + 8.958\log B}^{y_2}$$

$$+2.641 \qquad +2.641$$

$$32.641 = 8.958\log B$$

$$\div 8.958 \qquad \div 8.958$$

$$3.64378 = \log_{10} B$$

$$10^{3.64378} = B$$

$B = 4403$ butterfly observations