8.1 Understanding Logarithms

Using the log key on your calculator, determine the following:

1)	log 1 0	log 10 1	log 100 2	log 1000 3	log 10000 4
2)	log 1 0	log 0.1 −1	log 0.01 −2	log 0.001 −3	log 0.0001 −4
3)	log 0.126 −.89	log 1.26 0.10	log 12.6 1.10	log 126 2.10	log 1260 3.11
4)	log 1 0	log 2 0.3	log 3 0.47	log 4 0.6	log 5 0.7
5)	log 0 np	log (−1) np	log (−2) np	log (−3) np	log (−10) np.

What conclusions can you make about the log operation? What patterns do you observe?

you can't do log of a negative or 0
- as A increases, then logA increases
if A is 10 × bigger, then logA is 1 unit bigger.

In general, we can say that log $A = B$ means $10^B = A$

Thus log 1000 = 3 because $10^3 = 1000$

log 0.000 000 000 1 = −10 because $10^{-10} = 0.000\ 000\ 000\ 1$

log 2 = 0.3 because $10^{.3} \doteq 2$

The logarithms above are sometimes called common logarithms because they have a **base** of 10. In practice, if the base is 10, we do not write it down. Thus log A really means $\log_{10} A$. The base of the logarithm is not limited to 10, and can be defined with any positive base.

What is $\log_2 32$? This is read log, base 2, of 32 and means to find what power of 2 gives 32. Or in other words, the equations

$\log_2 32 = 5$

$$\log_2 32 = A \qquad \text{and} \qquad 2^A = 32 \qquad \text{are asking the same thing.}$$

Logarithmic form — Exponential form

So, $3 = \log_2 8$ means the same as $\log_2 8 = 3$.

Thus $\log_2 8$ is the exponent necessary in order to write 8 as a power of 2. Similarly, $\log_3 81$ is the exponent necessary in order to write 81 as a power of 3.

More generally, $\log_2 x$ is the exponent necessary in order to write x as a power of 2.

In general, $y = \log_a x$ is the exponent necessary in order to write x as a power of a.
Alternately, y is the exponent you need on a base of a to give an answer of x $a^y = x$

> Always remember: a logarithm is an exponent.

Equations written in exponential form can also be written in logarithmic form and vice versa.

Exponential Form		Logarithmic Form	
B can't be 1 $B > 0$ $A = B^x$	$\Leftrightarrow$	$\log_B A = x$	Restrictions: $B > 0$, $B \neq 1$, $A > 0$

It is extremely important to be aware of the restrictions on the above statement. $\log_3 9 = 2$

Convert to exponential form:			Convert to logarithmic form:		
$\log_a x = b$	$\Leftrightarrow$	$a^b = x$	$a^c = b$	$\Leftrightarrow$	$\log_a b = c$
$\log_3 x = 9$	$\Leftrightarrow$	$3^9 = x$	$5^{3m} = 8$	$\Leftrightarrow$	$\log_5 8 = 3m$

Examples

1) Express $5^4 = 625$ in logarithmic form $\log_5 625 = 4$	2) Express $\log_3 81 = 4$ in exponential form. $3^4 = 81$
3) Express $4^{-3} = \frac{1}{64}$ in logarithmic form. $\log_4 \frac{1}{64} = -3$	4) Evaluate $\log_3 3^5 = x$ $3^x = 3^5$ $\quad$ 5
5) Evaluate $\log_6 \sqrt{6} = x$ $6^x = \sqrt{6}$ $\quad$ $\frac{1}{2}$	6) Rewrite $y = 2^x$ as a logarithmic statement. $\log_2 y = x$

7) Evaluate : $\log_B B = 1$ $\quad$ $\log_B B^5 = 5$ $\quad$ $\log_B B^{13} = 13$ $\quad$ $\log_B B^x = x$

3^{-1} = fraction $\frac{1}{3}$

8) Determine $\log_B 1$ = 0 because anything^0 = 1	9) $\log_B 0 =$ not possible. $B^x = 0$ there is no x that works.
10) Determine x if $\log_3 x = -5$ $x = 3^{-5} = \frac{1}{243}$ * a log can	11) $\log_2(-16) = x$ $2^x = -16$ not possible. = a negative, but you can't do log of negative
12) $\log_{\frac{1}{2}} 8 = x$ $\left(\frac{1}{2}\right)^x = 8$ $(2^{-1})^x = 2^3$ $2^{-x} = 2^3$ $x = -3$	13) $\log_5 \frac{1}{25} = x$ $5^x = \frac{1}{25} = 5^{-2}$ $\boxed{x = -2}$
14) Determine x if $\log_x 36 = 2$ $x^2 = 36$ $x = 6$	15) Determine x if $\log_2 \underline{0.125} = x$ $\log_2 \frac{1}{8} = x \Rightarrow 2^x = \frac{1}{8}$ $\boxed{x = -3}$
16) $\log_2(\log_4 16)$ $\log_2 2 = 1$	17) $\log_3(\log_2(\log_{10} 10^8))$ $\log_3(\log_2(8))$ $\log_3(3) = 1$

18)	$\log_{10} 10^{0.4197}$ = .4197	$\log_3 3^{17}$ = 17	$\log_9 9^{0.123}$ = 0.123 $\log_a a^b = b$
19)	$10^{\log 100}$ = 100	$2^{\log_2 32}$ = 32	$7^{\log_7 49}$ = 49 $a^{\log_a b} = b$

Example 2. The formula for the Richter magnitude, M, of an earthquake is $M = \log \frac{A}{A_0}$, where A is the amplitude of the ground motion and A_0 is the amplitude of a standard earthquake. (A "standard" earthquake has amplitude of 1 micron and a magnitude of 0). The largest measured earthquake struck Chile in 1960, and measured 9.5 on the Richter scale. How many times as great was the seismic shaking on the Chilean earthquake than the 1949 Haida Gwaii earthquake, which had a magnitude of 8.1 on the Richter scale?

Chile earthquake = 9.5

H.G = 8.1

1.4 units on Richter Scale

$10^{1.4}$ = 25.12 x as great

150 000 x more powerful

log (150 000) = 5.2

our recent EQ = 4.3

The relationship between exponential and logarithmic functions can be seen by looking at $y = 2^x$ and $x = 2^y$ (This is the inverse of $y = 2^x$, and can also be written in logarithmic form as $y = \log_2 x$)

$y = 2^x$

x	-3	-2	-1	0	1	2	3
y	$1/8$	$1/4$	$1/2$	1	2	4	8

$x = 2^y$ or $y = \log_2 x$

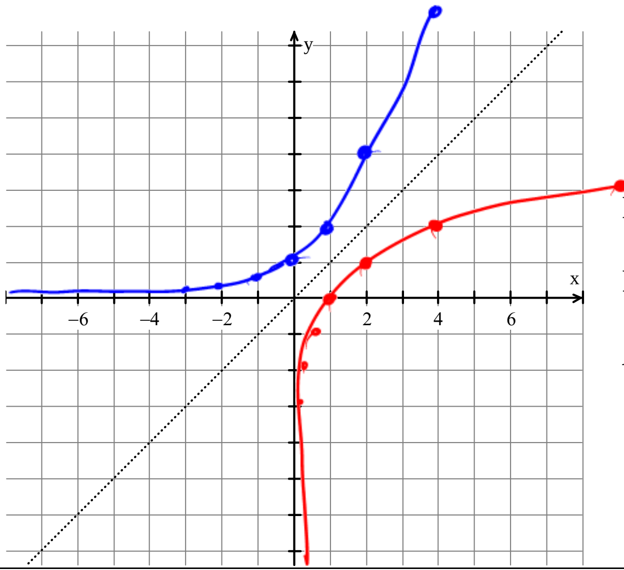
x	$-1/8$	$-1/4$	$-1/2$	1	2	4	8
y	-3	-2	-1	0	1	2	3

Exponential Function
$y = 2^x$

Domain $x \in \mathbb{R}$

Range $y > 0$

Asymptote $y = 0$



Logarithmic Function
$y = \log_2 x$

Domain $x > 0$

Range $y \in \mathbb{R}$

Asymptote $x = 0$

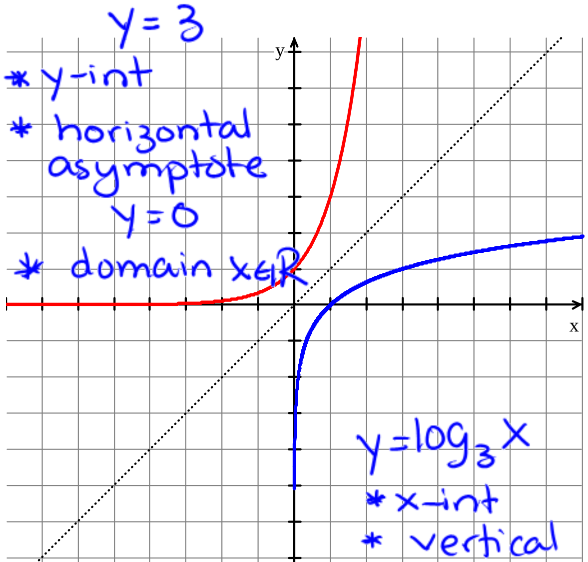
In general, we can then say that the inverse of the exponential function $y = a^x$ $(a > 0, a \neq 1)$ is $x = a^y$ or, in logarithmic form, $y = \log_a x$. Thus the inverse of $y = 3^x$ is $y = \log_3 x$ and the inverse of $y = \log_{10} x$ is $y = 10^x$. Give the properties of the graphs of logarithmic functions graphed below.

$y = 3^x$

* y-int

* horizontal asymptote $y = 0$

* domain $x \in \mathbb{R}$



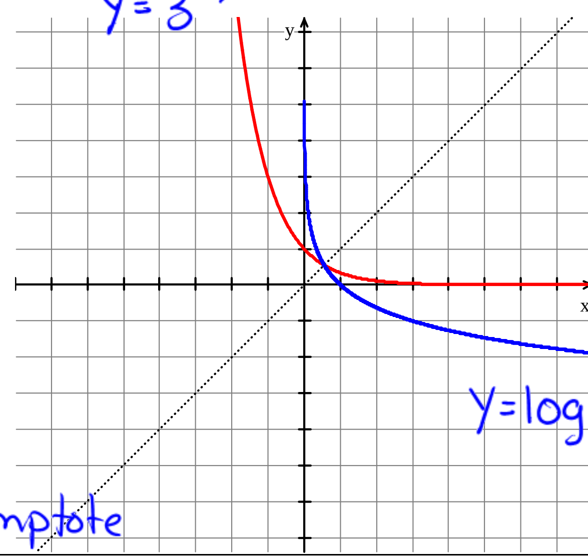
$y = \log_3 x$

* x-int

* vertical asymptote at $x = 0$

* range $y \in \mathbb{R}$

$y = 3^{-x}$



$y = \log_3 -x$