11.1B Permutations With Constraints

Example 1: Albane, Beau, Charlotte, Duncan and Eleanor go to watch a movie and sit in 5 adjacent seats. In how many ways can this be done under each of the following conditions?

a) without restrictions

$$5!$$

$$\underline{5} \ \underline{4} \ \underline{3} \ \underline{2} \ \underline{1}$$

b) if Charlotte insists on sitting in the middle

$$\underline{4} \ \underline{3} \ \underline{\frac{1}{C}} \ \underline{2} \ \underline{1} \qquad 4!$$

c) if the three girls sit between Beau and Duncan

$$\underline{2} \ \underline{} \ \underline{} \ \underline{} \ \underline{1} \quad \leftarrow \text{Boys}$$
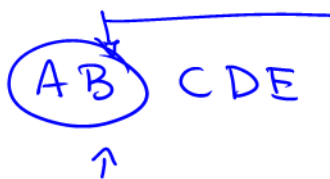$$ \quad 3 \ 2 \ 1 \quad \leftarrow \text{Girls}$$

Boys: 2! ways to arrange their seats

Girls 3! ways to arrange their seats.

$$\boxed{2! \times 3!}$$

d) if Albane sits next to Beau

(A B) C D E

↑

4 people, 4!

2! ways to arrange AB

$$\boxed{2! \times 4!}$$ ← how many ways can B and A be together.

e) if Albane refuses to sit next to Beau

$$5! \ - \ 2! \times 4! \ = \ 72$$

↗ all ways

↑ ways that AB are together

↑ ways that AB are not together.

Example 2: In how many ways can four adults and five children be arranged in a single line under each condition? ABCD ↙   ↪ 12345

a) without restrictions	b) if children and adults are alternated
9!	$\dfrac{5}{C}\ \dfrac{4}{A}\ \dfrac{4}{C}\ \dfrac{3}{A}\ \dfrac{3}{C}\ \dfrac{2}{A}\ \dfrac{2}{C}\ \dfrac{1}{A}\ \dfrac{1}{C}$ Children: 5! Adults : 4! 5! × 4!
c) if the adults are all together and the children are all together (ABCD) (12345) 2 groups : 2! within adults : 4! ⟨2! × 4! × 5!⟩ children : 5!	d) if the adults are all together (ABCD)(1)(2)(3)(4)(5) 6 things : 6! within adult group : 4! 6! × 4!

Example 3: How many ways can 4 boys and 3 girls sit in a row if: ABCD   123

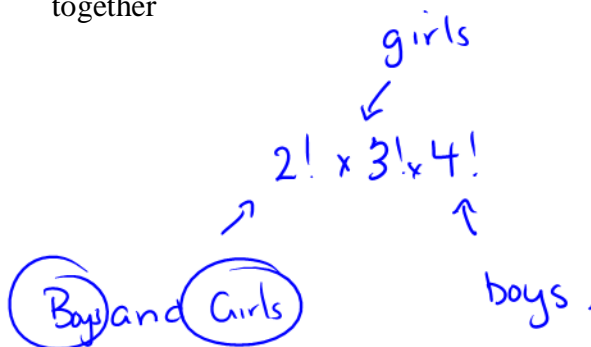| a) there must be a boy at the beginning and end of the row	b) all the boys sit together and all the girls sit together
$\underline{4}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{3}$ Ends : $_4P_2 = \dfrac{4!}{(4-2)!} = \dfrac{4!}{2!} = \dfrac{4\times3}{=}$ middle 5! 4×3× 5!	girls ↓ 2! × 3! × 4! ↗ ↑ (Boys) and (Girls) boys.

Example 4: How many different 3 digit odd numbers can you make using the digits 1 to 6 if the numbers must be less than 700? No digits are repeated.

$$\underline{5}\ \underline{4}\ \underline{3}$$
$$\uparrow$$
$$1, 3\ or\ 5$$

60

Example 5: Zoe has 6 maple, 5 fir and 3 pine trees that she wishes to plant in a row. The trees from the same species appear to be identical.

a) How many possible arrangements are there of the trees?

MMMMMM FFFFF PPP $= \dfrac{14!}{6!\,5!\,3!}$

b) If the first and last tree must be maple, how many arrangements are there?

M̶M̶MMMM F FF FF P P P

$\uparrow$
One M gone for front
One M gone for back

$\dfrac{12!}{4!\,5!\,3!}$

p524
#1-11,
14 - 23, 25